AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

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Candidate signature _____

# A-level MATHEMATICS

## Unit Mechanics 5

Friday 16 June 2017 Afternoon Time allowed: 1 hour 30 minutes

**Materials**

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

| For Examiner's Use | |
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| Question | Mark |
| 1 | |
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| **TOTAL** | |

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\ \text{m s}^{-2}$, unless stated otherwise.

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

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Answer **all** questions.

Answer each question in the space provided for that question.

**1** A simple pendulum consists of a light string of length $2.45$ metres and a small spherical bob. The pendulum is released from rest with the string taut. When it is released, the angle between the string and the vertical is $\frac{\pi}{10}$.

**(a)** Find the period of the motion of the pendulum.

**[2 marks]**

**(b)** Find the average speed of the spherical bob during one complete oscillation.

**[3 marks]**

**(c)** Find the angle between the string and the vertical when the speed of the spherical bob is $1.2 \text{ m s}^{-1}$.

**[5 marks]**

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**2** The points $A$ and $B$ are on a vertical line and $2$ metres apart, with $A$ above $B$.

Two identical elastic springs have natural length $0.5$ metres and modulus of elasticity $49\ \text{N}$. One end of each spring is attached to a particle of mass $0.4\ \text{kg}$. The other ends of the springs are attached to the points $A$ and $B$.

**(a)** Find the distance of the particle from $A$ when it is in equilibrium.

**[4 marks]**

**(b)** The particle is then pulled down below its equilibrium position and released. Show that the period of the subsequent motion is $\frac{\pi\sqrt{10}}{35}$ seconds.

**[5 marks]**

**(c)** Given that the particle is released from rest at a point $0.05$ metres below the equilibrium point:

**(i)** find the maximum speed of the particle;

**[2 marks]**

**(ii)** find the speed of the particle when it is midway between the equilibrium position and the point of release.

**[3 marks]**

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**3** A smooth wire is in the shape of the curve $y = 4 - x^2$. The wire is fixed in a vertical plane with the $y$-axis vertical. A small ring, of mass $m$, is threaded onto this wire. An elastic string has modulus of elasticity $4mg$ and natural length $1$. One end of the string is fixed to the point with coordinates $(0, 0)$ and the other to the ring.

Gravitational potential energy is taken to be zero at the level of the $x$-axis.

**(a)** Show that the total potential energy, $V$, of this system is given by

$$V = mg\left(38 - 15x^2 + 2x^4 - 4\sqrt{16 - 7x^2 + x^4}\right)$$

where $x$ is the $x$-coordinate of the position of the ring.

**[5 marks]**

**(b)** Find $\frac{\mathrm{d}V}{\mathrm{d}x}$ and show that the system has a position of equilibrium when $x = 2$.

**[4 marks]**

**(c)** Determine the nature of this equilibrium position.

**[3 marks]**

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**4** A particle moves on a curve and at time $t$ has polar coordinates $(r, \theta)$, where $r = \sin\theta$ and $\theta = 2t$.

**(a)** Show that, when $r = 1$, the transverse component of the acceleration of the particle is zero.

**[5 marks]**

**(b)** Find an expression for the times when the radial component of the acceleration is zero.

**[4 marks]**

**(c)** Show that, when the radial component of the acceleration is zero, the transverse component of the velocity is also zero.

**[2 marks]**

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**5** A particle, of mass $m$ kg, is attached to one end of an elastic string which has modulus of elasticity $2.5m$ newtons. The natural length of the string is $0.5$ metres. The other end of the string is attached to a fixed point $O$.

There is a container of fluid below $O$, where the surface of the fluid is $0.5$ metres below $O$. The particle is released from rest at a point vertically below $O$, on the surface of the fluid. As it moves through the fluid, the particle experiences a resistance force of magnitude $2mv$ newtons, where $v$ is the speed of the particle.

At time $t$ seconds after the particle is released from rest, the displacement of the particle below the surface of the fluid is $x$ metres.

Find the maximum length of the elastic string during the subsequent motion.

**[15 marks]**

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**6** A spacecraft is descending towards the surface of a planet and fires its rockets to prepare for landing. The initial mass of the spacecraft, including fuel, is $M$ kg and $10\%$ of this is fuel. The fuel is expelled at a rate of $\lambda$ kg s$^{-1}$ and at a speed of $U$ m s$^{-1}$ relative to the spacecraft.

All the motion takes place near to the surface of the planet and the acceleration due to gravity on the planet is a constant $g$ m s$^{-2}$ throughout the motion. When the rockets are fired, the speed of the space craft is $\frac{U}{20}$ m s$^{-1}$. At time $t$ seconds after the rockets start to fire, the velocity of the space craft is $v$ m s$^{-1}$.

Assume that no air-resistance forces act on the spacecraft as it moves.

**(a)** Show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = g - \frac{\lambda U}{M - \lambda t}$$

**[5 marks]**

**(b)** Given that the spacecraft slows down at the time that the rockets are fired, show that

$$U > \frac{Mg}{\lambda}$$

**[3 marks]**

**(c)** Given that $U = \frac{3Mg}{2\lambda}$, find the speed of the spacecraft when all the fuel has been used up, giving your answer in terms of $g$, $M$ and $\lambda$.

**[5 marks]**

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**END OF QUESTIONS**

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**ANSWER IN THE SPACES PROVIDED**

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